Fatigue of metallic materials

(Libor Pantělejev)

1) History of fatigue
2) Loading modes
3) Stages of fatigue life
4) Initiation of fatigue cracks
5) Life time curves
6) Influence of mean stress
7) Lifetime prediction
1) History of fatigue

1837 – Albert – first results of fatigue test (Clausthal)

1842 – Rankine, York – fatigue strength of railway’s axle (London)


1854 – Braithwaite – term “fatigue” was mentioned first time (description of a number service fatigue failures – water pumps, propeller shafts, levers, cranes etc.)

1858 – Wöhler – wide spread measurement of loading during service of four-wheeled and six-wheeled freight and passenger cars (results published in 1860)
1870 – “Wöhler’s law“—“Material can be induced to fail by many repetitions of stresses, all of which are lower than the static strength. The stress amplitudes are decisive for the destruction of the cohesion of the material. The maximum stress is of influence only in so far as the higher it is, the lower are the stress amplitudes which lead to failure“.

Wöhler therefore stated the stress amplitudes to be the most important parameter for fatigue life, but a tensile mean stress also to have a detrimental influence.

Followers: Smith, Haigh, Palmgren, Miner, Paris, Klesnil and others
2) Fatigue test at different loading modes:

**Load (stress) control regime:** \( \sigma_a = \text{const.} \)

high-cycle fatigue (Wöhler, Basquin)

**Testing machine:**

- resonant testing machine
- servo-hydraulic testing machine
- ultrasonic testing machine (ultra-high cycle fatigue)

**Strain control regime:** \( \varepsilon_a = \text{const.} \)

Low-cycle fatigue (Manson – Coffin)

**Testing machine:**

- servo-hydraulic testing machine
Basic characteristics of loading cycle

- **resonant testing machines** are working with sinusoidal loading cycle
- **servo-hydraulic testing machines** allow selection of type of the loading cycle e.g. triangular, trapezoidal, saw-like etc.

\[ \Delta \sigma = 2 \sigma_a \]

- \( \Delta \sigma \) – stress range \((\Delta \sigma = 2\sigma_a)\)
- \( \sigma_a \) – stress amplitude
- \( \sigma_{\text{max}} \) – maximum stress
- \( \sigma_{\text{min}} \) – minimum stress
- \( \sigma_m \) – mean stress

\[
\begin{align*}
\sigma_a &= \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \\
\sigma_m &= \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}
\end{align*}
\]
Strain controlled regime:

\[ \Delta \varepsilon \] – strain range \((\Delta \varepsilon = 2\varepsilon_a)\)

\(\varepsilon_a\) – strain amplitude

\(\varepsilon_{\text{max}}\) – maximal strain

\(\varepsilon_{\text{min}}\) – minimal strain

\(\varepsilon_m\) – mean strain

Testing is conducted usually in symmetrical loading cycle.
Parameters of loading cycle asymmetry

Stress ratio

\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]

\[ P = \frac{\sigma_{\text{max}}}{\sigma_a} \]

\[ A = \frac{\sigma_a}{\sigma_m} \]

Amplitude ratio

\[ P = \frac{2}{1 - R} \]

\[ A = \frac{1 - R}{1 + R} \]

\[ A = \frac{1}{P - 1} \]

Symmetrical cycle:

\( R = -1, \ P = 1, \) can not be described via parameter A – problem of singularity (division by zero)

Repeated cycle:

\( R = 0, \ P = 2, \ A = 1 \)
\[ P = \frac{\sigma_{\text{max}}}{\sigma_a} \]

\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]

![Diagram of cyclic loading types](image)

- **Pulsating cycle in tension**: \( P > 2 \), \( 0 < R < 1 \)
- **Repeated cycle in tension**: \( P = 2 \), \( R = 0 \)
- **Symmetrical cycle**: \( P = 1 \), \( R = -1 \)
- **Pulsating cycle in compression**: \( P < 0 \), \( 1 < R < +\infty \)
- **Repeated cycle in compression**: \( P = 0 \), \( R = -\infty \)
how to reach full load

Resonant testing machine:
- loading frequency is given by stiffness of the system (testing machine – tested specimen) – there is possibility partly to change the loading frequency by operator (add or remove of additional weight)
- full load is not reached immediately – loading ramp is inherent property of apparatus

Servo-hydraulic testing machine:
- the loading ramp/full load can be reached immediately
- loading ramp: either number of cycles to reach full load or selection of the loading frequency (in the range given by testing machine).
3) Stages of fatigue life

- Changes of mechanical properties
  cyclic softening/hardening

- Crack initiation
  - surface layers
  - inhomogeneity

- Crack propagation

Final fracture
Changes of mechanical properties

Measurement of cyclic stress-strain response (hysteresis loop)

„clip-on“ extensometer:
- X-Y plotter
- digital reading - PC – more accurate in comparison with

Reading of unidirectional strain (elongation):
- inductive transducer (inductor and movable core) – direct method (digital reading - PC)
- Displacement of hysteresis loops (hysteresis loop is sliding along axis X)
Softening/hardening curves

Cyclic stress-strain curve

Manifestation of cyclic hardening

Manifestation of cyclic softening
Cyclic softening

Δσ = const.

Cyclic hardening

Δσ = const.

Cu

σ_a = const.
Cyclic stress-strain curve (CSSC)

Description of the cyclic plastic response of material

Available for both type of loading (load/strain control):

- from the saturated states
  \( \frac{\varepsilon_{ap,sat}}{\sigma_{a,sat}} \)
- for \( \frac{1}{2} N_f \) in the case of no saturation
  \( \frac{\varepsilon_{ap,50\%}}{\sigma_{a,50\%}} \)

Power law form is often used for description of CSSC:

\[
\sigma_a = K'(\varepsilon_{ap})^{n'}
\]

- \( K' \) - cyclic hardening coefficient
- \( n' \) - cyclic hardening exponent

\( \sigma_a \) - stress
\( \varepsilon_{ap} \) - plastic strain
\( R = -1 \): Fatigue fracture
\( R = 0 \): Ductile (creep) fracture
Development of specific dislocation (sub)structure during loading

Stacking fault energy

FCC metals

Persistent slip bands

Stacking fault energy $\gamma$ (J.m$^{-2}$)

Au ........ 0,03
Cu ........ 0,04
Al ........ 0,14
Ni ...........0,2
Initiation of fatigue cracks

**Homogeneous materials:**
- surface initiation (specimen, component)

**Inhomogeneous materials:**
- interface between hardened surface layer and “softer” matrix (e.g. nitrocarburized steel etc.)
- Interface between inclusion and matrix (influence of inclusions – purity of pure metals and alloys)

For materials with prior cracks – it is assumed that initiation phase of fatigue process is missing

Basic type of crack initiation is focused to fatigue bands
Crack initiation in persistent slip bands

Fatigue bands – manifestation of cyclic plastic deformation
- evolution will start at the end of the stage of mechanical properties changes
- progressive evolution of fatigue bands with increasing Nf
- surface relief evolution – intrusion/extrusion, intensity of bands is dependent on parameters of loading ($\sigma_m = 0$, $\sigma_m > 0$)

$\sigma_a = 137$ MPa, $N_f \approx 1.1 \times 10^6$

Loading direction

70Cu-30Zn
Persistent slip bands

- $R = -1$
  - $\sigma_a = 255$ MPa
- $R = 0.1$
  - $\sigma_a = 160$ MPa
Stage of crack initiation:

\[ N_0 \] – number of cycle required for crack initiation
\[ N_f \] – number of cycles to failure

Relative number of cycles \( N_0/N_f \)

\( N_0/N_f \) is dependent on:

a) Loading amplitude and asymmetry of cycle (\( N_0/N_f \) decreasing with increasing amplitude of loading)

b) Geometry of specimen (structural parts) – influence of notches (stres concentrators)

c) Microstructure (inclusions, hard locations)

d) Quality / roughness of surface layer (roughness, residual stresses, contact pitting, surface hardening)

e) Environment (corrosion pittings)
Fatigue fracture

Piston of chipping hammer

Overload

Crack propagation - striations

origin
fatigue crack propagation
Overload fracture
Initiation site
Broken blade of turbocharger
Manson – Coffin curve

\[ \varepsilon_{ap} = \varepsilon'_f \left(2N_f \right)^c \]

fatigue ductility exponent

fatigue ductility coefficient

\[ \varepsilon_{at} = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma'_f}{E} \left(2N_f \right)^b + \varepsilon'_f \left(2N_f \right)^c \]
(S-N curve)

A517 steel, $\sigma_u = 820$ MPa
- failure
- no failure, test stopped

Permanent fatigue limit

Fatigue limit for given $N_f$
Wöhler: \[ \sigma_a = AN_f^B \]

Basquin: \[ \sigma_a = \sigma_f' \left(2N_f \right)^b \]

\[ A = 2^b \sigma_f', B = b \]

fatigue strength coefficient

fatigue strength exponent
Fatigue life curves obtained for different loading regime ($\sigma_a = \text{const.}$, $\varepsilon_{ap} = \text{const.}$)

It is possible to convert between one another through cyclic stress strain curve (CSSC).

\[
\sigma_a = K'(\varepsilon_{ap})^{n'}
\]

\[
\sigma_a = \sigma'_f \left(2N_f\right)^b
\]

\[
\varepsilon_{ap} = \varepsilon'_f \left(2N_f\right)^c
\]

\[
K' = \frac{\sigma'_f}{\varepsilon'_f^{n'}}
\]

\[
b = n'c
\]

Parameters of fatigue life curves are not independent:
5) Influence of mean stress

Stress amplitude [MPa]

Symmetrical cycle

$\sigma_m = 414$ MPa

$\sigma_{max}$

$\sigma_{min}$

$\sigma_a$

$\Delta \sigma$

$N_f$, Cycles to failure
Constant-life diagram

\[ R_m = \sigma_u \]

Ultimate tensile strength

\[ \sigma_m, \text{ ksi} \]

\[ \sigma_a \text{ Stress amplitude [MPa]} \]

\[ \sigma_m \text{ Mean stress [MPa]} \]

\[ N_f = 10^4 \text{ cycles} \]

\[ 10^5, 10^6, 10^7, 5 \times 10^8 \]

\[ \sigma_a = \sigma_{ar} \]
Normalized Amplitude-Mean Stress Diagram

\[
\frac{\sigma_a}{\sigma_{ar}} = 1
\]

\[
\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{R_m}\right)^2 = 1
\]

\[
\frac{\sigma_a}{\sigma_{ar}} = \frac{\sigma_m}{R_m} = 1
\]

7075 - T6 Al

Normalized stress amplitude vs. mean stress [MPa]
Goodman

\[ \frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1 \]

\( \sigma_a \) – stress amplitude of common cycle
\( \sigma_{ar} \) – stress amplitude of symmetrical (regular) cycle
\( \sigma_u \) – ultimate tensile strength
\( \sigma_e \) – yield strength

Gerber

\[ \frac{\sigma_a}{\sigma_{ar}} + \left( \frac{\sigma_m}{\sigma_u} \right)^2 = 1 \]

Soderberg

\[ \frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_e} = 1 \]
Wöhler:

\[ \sigma_{ar} = AN_f^B \]

Basquin:

\[ \sigma_{ar} = \sigma'_f \left(2N_f\right)^b \]

Goodman:

\[ \frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1 \]

\[ \sigma_a = A \left(1 - \frac{\sigma_m}{\sigma_u}\right)\left(N_f\right)^B \]

\[ \sigma_a = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}} \left(2N_f\right)^b \]

For more accurate description \( \sigma'_f \) is used in place of \( \sigma_u \)

\[ \sigma_a = \left(\sigma'_f - \sigma_m\right)\left(2N_f\right)^b \]
The Palmgren-Miner Rule

Failure is expected when

$$\frac{N_1}{N_f^1} + \frac{N_2}{N_f^2} + \ldots = \sum_{j=1}^{k} \frac{N_j}{N_f^j} = 1$$

$$B = \frac{1}{\frac{N_1}{N_f^1} + \frac{N_2}{N_f^2} + \frac{N_3}{N_f^3}}$$

Cycles to failure

Fraction of the life

Expected Failure
Literature:

